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On Approximately Convex Functions

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Abstract

Following a recent work of Chademan-Mirzapour (1995&1999). we generalize the classical theorems of Jensen, Bernstein-Doetsch, Ostrowski, Blumberg-Sierpinski and Mehdi on approximately midconvex functions in real vactor spaces to approximately midconvex functions on topological groups. We define also ?ƵWright-convexity in topological groups and prove a theorem on it.

Keywords: *approximately convex functions, Bernstein-Doetsch theorem, Jensen's theorem, Ostrowski's theorem, Blumberg-Sierpinski theorem, Mehdi's theorem,Wright-convex function.*

1. Introduction

In this article, our main idea has been based on papers of Chademan and Mirzapour (1995&1999). First we studied approximately convex functions on Euclidean space and real vector spaces (Hyers and Ulam, 1952; Ng and Nikodem, 1993). Then, we extended this concept to topological groups, and generalized the important theorems of this new concept (Chademan and Mirzapour, 1995 & 1999).

Let R^n be the *n*-dimensional Euclidean space. A function *f* defined on an open convex subset *S* of R^n is called ?ὲconvex (Hyers and Ulam, 1952), if:

f(?Nx + (1- ?N)y) ≤ ?N*f*(*x*) +(1- ?N)*f*(*y*) + ?N (1)

for all *x*, *y* ∈ *S* and ?Ç∈ [0, 1]. Hyers and Ulam (1952) proved that for an ?N-convex function *f* on *S*, there exists a convex function ?N on *S* such that $|f(x) - ?Ž(x)| \le \frac{n^2+3n}{4n+4}$?‡ for *x* ∈ *S*.

In 1993, Ng and Nikodem, using similar definitions for ?Î-convex and ?-midconvex (i.e. ? = $\frac{1}{2}$), and also by defining the ? -Wright-convex function on an open convex subset of a real vector space *X* to conclude two important theorems of Bernstein- Doetsch and Ostrowski type and many theorems about ?Ì-Wright-convex functions (Ng and Nikodem, 1993).

Chademan and Mirzapour, propounded definitions of midconvex functions in an open midconvex subset [for the definition of midconvex subset see (Chademan and Mirzapour, 1999) or section 2 of this article] of topological groups and proved the Jensen's, Bernstein-Doetsch, Ostrowski's and Blumberg-Sierpinski theorems.

Morassaei and Alizadeh (1998) proved the Bernstein-Doetsch and Ostrowski's theorems on approximately convex functions for topological groups.

In this article, we define **upper ?ó-semicontinuity** of a function and use a suitable definition of ? -midconvex, ? -convex, and ? -Wright-convex functions on topological groups to show the following theorems:

Theorem 1. A globally ?Q-midconvex functions, on a midconvex open subset of a root-approximable topological group is 2?c-convex if it is bounded from above on some neighborhood of a point.

Theorem 2. Let ?" be a midconvex open subset in an abelian root-approximable locally compact topological group G endowed with a left-invariant Haar measure ?Â and *f*: ?Â → **R** be a globally ?Â-midconvex function. If *f* is bounded from above on a set E ⊂ ?Ô with 0 < ?é(E) < ∞, then *f* is 2?æ-convex.

Theorem 3. Let ?l be an open midconvex subset of a locally compact topological group G and *f*: ?õ → **R** be a globally ?õ-midconvex function and Haar measurable, then *f* is 2?~-convex.

Theorem 4. Let ? be an open midconvex set in an abelian root-approximable group G, and *f*: ?• → **R** a globally ?•-midconvex function. If *f* is bounded above on a subset of ? of the second Baire category, then *f* is 2?¢convex.

Theorem 5. Let ?+ be an open midconvex set in root-approximable abelian group G. If *f*: ?´ → **R** is **g**lobally ?´-Wright-convex and locally bounded from below at a point a ∈ ?‡, then *f* is 2 ?.-convex.

2. Definitions and Preliminaries

Let G be a (nondiscrete) topological group, not necessarily abelian, with the identity element *e*, and let ?1 ⊂ G be an open set and ?1 be a nonnegative constant and let Φ denote the filter of the neighborhoods of *e*. Assume *f* be a real-valued function on ?C In analogy with Chademan and Mirzapour, (Chademan and Mirzapour, 1995 & 1999), we present the following definitions, only with added suffix 2?µto the definitions.

Definition 1. *f* is called *globally* ?g*midconvex* in ?gif

$$2f\,(a) \le f\,(az) + f\,(az^{-1}) + 2? \qquad (2)$$

for all *a*,*z* such that *a*, *az*, az^{-1} ∈ ?‡ This inequality is called the ?º-midconvex inequality.

Definition 2. *f* is called *locally* ?N*midconvex* at *a* ∈ ?‡, if there exists an open symmetric set $V \in \Phi$ such that ?=midconvex inequality holds for all $z \in V$.

Definition 3. *f* is called *sequentially* ?Ó*midconvex* at *a*, if it is locally ?Ó -midconvex in ?Æand there exists a symmetric open set $V \in \Phi$ such that $aV^2 \subseteq \Phi$ and for all z ∈ V , the following condition is satisfied

$$2f\,(az) \le f\,(a) + f\,(az^2) + 2?\backslash$$

A function f which is sequentially ?Nmidconvex at each point of ?N is called sequentially ?×midconvex in ?×

Lemma 1. f: ?0 $\rightarrow$ **R** is sequentialy ?0-midconvex at a point a if and only if there exists an open symmetric neighborhood $aV = aV^{-1}$ such that $aV^{2} \subseteq$?' and for every $y \in V$ the following condition is satisfied

$$\forall z \in V,\ 2f(ay) \leq f\,(ayz^{-1}) + f\,(ayz) + 2?l \qquad (3)$$

Definition 4. f is called *upper ?Psemicontinuous* at a, if for ?P> there exists a neighborhood aV, $V \in \Phi$, such that for all $z \in V$ the following inequality holds:

$$f\,(az) \leq f\,(a) + ?® ?®$$

Definition 5. f is called ? *-convex* at a if it is locally ? -midconvex and upper ?•semicontinuous at a. If f is ?•-convex at any point of ?•, f is said to be ? -convex on ? .

Definition 6 (Chademan, and Mirzapour,1999). An element $x \in G$ is said to be *root-approximable* if there exists a sequence $(x_n)_{n \geq}$ of elements G such that

$$\lim_{n?ý?‡} x_n \ ?p e, \qquad x_n^{2^n} \ ?p\, x.\ (n \ ?‡\, 0, 1, 2, ...)$$

If every element of G is root-approximable, we say G is a root-approximable group.

Definition 7 (Chademan, and Mirzapour, 1999). A subset E of the topological group G is called *right midconvex* if for every $x, y \in E$, there exists a $z \in G$ such that $x\,z \in E$ and $xz^2 = y$.

Lemma 2 (Morassaei and Alizadeh, 1998). Let f : ?Õ$\rightarrow$ **R** be a globally ?Èmidconvex and $x, a \in$ G be such that $\{x, xa, \ldots, xa^n\} \subset$?È for some $n \in$ **N**. Then

$$f\,(xa^{m}) \le (1-\frac{m}{n})\, f\,(x) + \frac{m}{n} f\,(xa^{n}) + mn(1-\frac{m}{n}) \text{ ?5} \quad (4)$$

for all positive $m \le n$.

Proof. This is proved by induction on n. For $n = 1$, the inequality is clear. Assume that (4) holds for $m = n - 2$, since f is globally ? -midconvex, we have

$$2f\,(xa^{n-1}) \le f\,(xa^{n-2}) + f\,(xa^{n}) + 2 \text{ ?1}$$

and by induction

$$f\,(xa^{n-2}) \le \frac{1}{n-1} f\,(x) + \frac{n-2}{n-1} f\,(xa^{n-1}) + (n\text{-}2) \text{ ?}$$

therefore we get

$$f\,(xa^{n-1}) \le (1-\frac{n-1}{n}) f\,(x) + \frac{n-1}{n} f\,(xa^{n}) + (n\text{-}1)n\,(1-\frac{n-1}{n}) \text{ ?¨} \quad (5)$$

If for $1 \le m \le n - 1$ condition (4) holds, we get

$$f\,(xa^{m}) \le (1-\frac{m}{n-1}) f\,(x) + \frac{m}{n-1} f\,(xa^{n-1}) + m(n\text{-}1)\,(1-\frac{m}{n-1}) \text{ ?•.}$$

In view of condition (5); we have

$$f\,(xa^{m}) \le (1-\frac{m}{n}) f\,(x) + \frac{m}{n} f\,(xa^{n}) + mn(1-\frac{m}{n}) \text{ ?¾}$$

as required. This proves the lemma.

Proposition 1 (Chademan and Mirzapour, 1999; Morassaei and Alizadeh, 1998) (The generalization of Jensen's Theorem). Let ?–$\in G$ be an open midconvex subset and f : ?¨$\to$ **R** be a sequentially ?¨-midconvex function in a $\in$?œif

$$\text{Lim sup}_{x\to e} f(ax) < + \text{ ?ª}$$

Then f is 2?¼convex at a.

Corollary 1. The sequentially ?ë-midconvex function f : ?ë$\to$ **R** is 2?ýconvex if and only if it is locally bounded from above.

Remark 1 (Morassaei And Alizadeh, 1998). If G is an abelian root-approximable topological group and ?Ú⊂ G is an open midconvex set and f : ?c→ **R** is a globally ?Gmidconvex function, then

$$\forall x,y \in \text{?‡} \qquad f\left(\frac{x+y}{2}\right) \le \frac{1}{2} f(x) + f(y) + \text{?…}.$$

Remark 2. We note that there exist functions that are ?•convex but not convex. For example, the characteristic function ?‡$_{(-\infty,\,0]}$ is 1-convex but not convex.

3. The Generalization of Bernstein-Doetsch and Ostrowski's Theorems

In this section, we extend Bernstein-Doetsch and Ostrowski's theorems. Their proofs are similar to Chademan and Mirzapour, (Chademan and Mirzapour, 1999).

Theorem 1 (Chademan, and Mirzapour, 1999). Let G be a root-approximable topological group, and ?n be midconvex subset of G. Assume that f : ?€→ **R** is globally ?€midconvex. If there exists a point $a \in \Omega$ and a neighborhood aV of a, $V \in \Phi$, such that f is bounded from above on aV, then f must be 2?Αconvex in ?n.

Proof. Without loss of generality it can be assumed that $e \in \Omega$. Suppose that $f \le$?f on V, $V \in \Phi$ and $V \subset$?f. We show that for every $y \in$?f, f is bounded from above on some neighborhood of y.
Let y be an arbitrary element of ?,. Since G is root-approximable, there exists a sequence $(y_n)_{n \ge}$ such that y_n ?‡$y^{\frac{1}{2^n}} \to e$. Hence, there exists an integer m such that $yy_m \in$?‡ Therefore we can take a chain $(V_i)_{1 \le i \le 2^m+1}$ of symmetric open sets of Φ with

$$V_{2^m+1} \text{ ?‡} V_{2^m} \text{ ?‡} \ldots \text{ ?Ô} V_2 \text{ ?‡} V_1$$

and the properties :

$$\begin{cases} (1) & V_1^{2^m+1} \subset V \\ (2) & y_m^{-1}V_i \subset V_{i-1}\; y_m^{-1} \cap y_m^{-1} V_{i-1} \end{cases}, \qquad i = 2,3,\ldots,2^m+1 \tag{6}$$

We prove that $f \le \boldsymbol{C}$ on the neighborhood $(V_{2^m+1})y$, for

$$\boldsymbol{C} = (1- \frac{2^m}{2^m+1})\, M + \frac{2^m}{2^m+1} f(yy_m) + 2^m(2^m+1)(1-\frac{2^m}{2^m+1})?0$$

Let $x \in V_{2^m+1}$ and $z = (y_m^{-1}x)^{2^m}$. By (6), there exists x_1 in V_{2^m} such that $y_m^{-1}x = x_1y_m^{-1}$. Therefore

$$z = (x_1\, y_m^{-1})^{2^m} = x_1(y_m^{-1}x_1)\ldots(y_m^{-1}x_1)\, y_m^{-1} = x_1\, (y_m^{-1}x_1)^{2^m-1}\, y_m^{-1}.$$

Also there exists $x_2 \in V_{2^m-1}$ such that $y_m^{-1}x_1 = x_2y_m^{-1}$. Thus

$$z = x_1(y_m^{-1}x_1)^{2^m-1}\, y_m^{-1} = x_1(x_2y_m^{-1})^{2^m-1}y_m^{-1} = x_1x_{2(}\, y_m^{-1}x_2)^{2^m-2})\, y_m^{-2}.$$

By repeating this process, for $1 \le i \le 2^m$, we get $x_i \in V_{2^m+1-i}$ satisfying

$$z = x_1x_2\ldots x_i(y_m^{-1}x_i)^{2^m-i}\, y_m^{-i}$$

For $i = 2^m$, we have

$$z = x_1\ldots x_{2^m}y^{-1},$$

therefore $zy \in V_1^{2^m}$ and by property (6),

$$xzy \in V_1^{2^m+1} \subseteq V.$$

By Lemma 2, we obtain

$$f(xy) \le (1-\frac{2^m}{2^m+1})f(xzy) + \frac{2^m}{2^m+1})f(yy_m) + 2^m(2^m+1)(1-\frac{2^m}{2^m+1})\ ?\le \boldsymbol{C}$$

The proof is complete, by proposition 1.

Proposition 2(Chademan, and Mirzapour,1999). Let G be a localty compact group and let ? be *a* left-invariant Hear measure. If *E* is a measurable set with 0 < ?N(*E*) < ∞, then each EE^{-1} and $E^{-1}E$ contains an open set from Φ.

Theorem 2 (Chademan, and Mirzapour,1999; Morassaei and Alizadeh, 1998). Let ?î be an open midconvex set in an abelian root-approximable locally compact nondiscrete group G, ?wa left-invariant Haar measure on G and f : ? → **R** a globally ? -midconvex function. If f is bounded from above on a set $E \subset$?%with positive Haar measure 0< μ(E) < ∞, then f must be 2 ?-convex on ? .

Proof. Since G is an abelian root-approximable locally compact nondiscrete group, then

$$H_1(\mathrm{E}) = \frac{E+E}{2} = ?\grave{a}\, \frac{?x\,?.y}{2}\; : x, y \,?\ddot{\imath} E\, ?\grave{a}$$

Therefore the interior of 2E is nonempty, by Proposition 2 and 0 < μ(E) < ∞. Assume $f \le$ M on E, hence

$$f\left(\frac{x+y}{2}\right) \le \frac{1}{2} f(x) + \frac{1}{2} f(y) + ?Æ?Æ+ ?Æ$$

for all $x,y \in E$. Then f is 2?'-convex on ?', by Theorem 1.

4 The Generalization of Blumberg-Sierpinski Theorem

In this section, we generalize the Blumberg-Sierpinski theorem on a approximately midconvex functions in locally compact groups. To show this theorem, we use the *modular function* Δ which is equal to 1 on compact groups (Hewitt and Ross, 1963).

Lemma 3. Let : ?Ë→ **R** be locally ?Ë-midconvex at a, and locally bounded from above at a, then f is locally bounded from below at a.

Proposition 3. Let f : ?ú→ **R** be sequentially ?ú-midconvex and δ-convex at $a \in \Omega$. Then f is 2?î-convex at a.

Proof. Since f is sequentially ?ü-midconvex at a, it is therefore locally 2?-midconvex at a. It is enough to show f is upper 2?-semicontinuous at a. Assume that f is not upper 2? -semicontinuous, then, there exists ? > such that for all open symmetric neighborhoods $aV = aV^{-1}$,

$$\exists\, y \in V;\ f(ay) > f(a) + 2?]+ ?] \qquad (7)$$

By induction, we show that

$$\forall\, W \in \Phi,\ \exists\, y \in W;\ f(ay) > f(a) + 2?ð+ 2^{n}?‡ \qquad (8)$$

The initial step is clear by (7). Assume (8) holds for n and $W \in \Phi$ is arbitrary. Since f is sequentially ?-midconvex at a, there exists an open symmetric set V in Φ such that $V^2 \subseteq W$ and for all $x \in V$,

$$2f(ax) - f(a) \leq f(ax^2) + 2?.$$

By induction, there should exist $y \in V$ such that

$$f(ay) > f(a) + 2? + 2^{n}?‡$$

so,

$$2f(ay) > 2f(a) + 4? + 2^{n+1}?‡$$

By (9), we have

$$2f(ay^2) \geq f(a) + 2?5+ 2^{n+1}?‡$$

This is a contradiction , since f is δ-convex at a,

Corollary 2. Let f : ?• → **R** be sequentially ?•-midconvex at a and locally bounded from above at a. Then f is 2?¡-convex at a.

Proof. By the hypothesis and Lemma 3, there exists an open symmetric set $U \in \Phi$ such that $2f(a) \leq f(ay) + f(ay^{-1}) +$?®and $\leq |f(ay)| \leq f(a) + M$. In other words, f is $2M$-convex at a. Thus f is ?¼convex at a, by proposition 3.

Proposition 4. Suppose G is a locally compat group and ?ëis an open subset of G. If f : ?ý → **R** is a Haar measurable function and sequentially ? -midconvex at $a \in \Omega$, then f is ? -convex at a.

In this theorem, similar to Theorem 1, the main idea is from (Chademan and Mirzapour,1999) and we have only added suffix ?<to the proof.

Proof. In view of Corollary 2 it is enough to show that f is bounded from above in a neighborhood of a. Assume W is an open symmetric neighborhood of e with compact closure such that $a\,\overline{W} \subseteq$?‡ The modular function Δ is a continuous homomorphism in G and has minimum $m >$ on $\overline{W}$ and

$$\mu(Az) = \Delta(z)?\mathrm{N}A)$$

for every $z \in$ G and ?Ç measurable set $A \subset G$ (Hewitt and Ross, 1963). Since f is sequentially ?-Nmidconvex at a, there exist open symmetric sets U, $V \in \Phi$ such that $U^2 \subseteq V$, $V^2 \subseteq W$ and

$$\forall y \in V,\ 2f(ay) \le f(a) + f(ay^2) + 2?\mathrm{M}$$

If f is not bounded from above near a, then for every integer $n \ge 1$, there exists an element y_n in U such that $f(ay_n) > n$. Since f is sequentially ?-Ümidconvex, by Lemma 1, we can give V such that $V^2 \subseteq W$ and

$$2f(ay) \le f(ayz^{-1}) + f(ayz) + 2?ß \qquad (10)$$

for every $z \in V$.

For any arbitrary x in U, put $z_n = x^{-1}y_n$. By replaceing y, z in (10) by y_n, z_n respectively, we get

$$2f(ay_n) \le f(ay_nz_n^{-1}) + f(ay_nz_n) + 2?$$
$$= f(ax) + f(ay_nx^{-1}y_n) + 2?8$$

Now, let $A_n = \{x \in W : f(ax) > n-2?\mathrm{H}\}$ and $B_n = \{x \in W : f(ax) > n-2?\mathrm{H}\}$ for every integer $n \ge 1$. Thus, A_n and B_n are measurable sets and if $x \in B_n ?\mathrm{I}\ U$, then

$$f(ay_nx^{-1}y_n) = f(ay_nz_n) = f(axz_n^2) > n - 2?S$$

So, $(B_n\ \mathrm{I}\ U)z_n^2 \subseteq A_n$. Consequently

$$\mu[\,(B_n\ \mathrm{I}\ U)z_n^2\,] = \Delta(z_n^2)?\mathrm{0}(B_n ?\mathrm{I}\ U) \le ?\mathrm{0}(A_n).$$

But, since $z_n^2 \in U^2$, we get

$$m^2?\ddagger(B_n ?\mathrm{I}\ U) \le \Delta(z_n^2)?\dagger(B_n ?\mathrm{I}\ U)\ ?\dagger??A_n).$$

Thus,

$$m^2?\ddagger(W)?\pm?(\ m^2)[?(A_n) + ?(\ B_n)$$
$$?0(m^2+1)[?0(A_n) + m^2(?0(W)-?0(U)).$$

Since U is open with positive measure α = ?(U), we have

$$?(A_n) ? \frac{?m^2}{1+m^2} > 0$$

which is independent of n. Now, $(A_n)_{n \geq 1}$ is a decreasing sequence and ?(A_1) ≤ $= \mu(W)$, then

$$\lim_{n\to\infty} ?(A_n) = ?(\bigcap_{n=1}^{\infty} A_n).$$

Therefore, $?(\bigcap_{n=1}^{\infty} A_n) \geq \frac{?m^2}{1+m^2}$. So, there exists $x \in \bigcap_{n?}^{?} A_n$ such that $f(ax) > n$ for any $n \in$ **N**. This contradicts to the Archimedean property of real numbers.

Theorem 3. Let ?' be an open midconvex subset of a locally compact topological group G and $f : ? \to$ **R** be a globally ?-midconvex Haar measurable function, then f is 2?-convex on.

5 The Generalization of Theorem

In 1964, Mehdi (Mehdi, 1964) proved the following theorem:
Theorem. Let H be a non-empty open convex set in a topological vector space E and let f be a function on H. If f is bounded from above on a second category Baire subset S of H, then f is 2? -convex on H.

In this section, we prove the above theorem on approximately midconvex functions in topological groups.

Proposition 5 (Kominek and Kuczma, 1989). Let G be a semitopological group. If the sets $A, B \subset G$ are of second category and, moreover, A has the Baire property, then AB^{-1} contains a non-empty open set.

Remark 3. If in the above Proposition, G is a topological group, then

$$\text{int}(AB) \neq \emptyset.$$

Theorem 4. Let Ω be an open midconvex set in an abelian root-approximable group *G*, and $f : \Omega \to \mathbf{R}$ be a globally ?×-midconvex function. If *f* is bounded from above on a second category Baire subset of Ω, then *f* is 2?è-convex .

Proof. Assume for $M \in R$ and for a second category Baire subset *E* of ?±, $f(x) \leq M$ for every $x \in E$. Put $F = \dfrac{E+E}{2}$, then, for every $z \in F$, there exist $x, y \in E$ such that $z = \dfrac{x+y}{2}$. So $f(z) = f(\dfrac{x+y}{2}) \leq \dfrac{1}{2} f(x)$ + $f(y)$ + ?ú≤ ?ú+ ?ú by Remark 1, i.e., *f* is bounded from above on *F*. If we take $A = B = E$ in Proposition 4, we get $int\, F = (\dfrac{E+E}{2}) \neq \varnothing$.

Now, put $U = int\ F$, therefore, *f* is bounded from above on *U*. Consequently *f* is 2? -convex, by the Bernsein-Doetsch theorem.

6 The Generalization of ?=Wright-convexity

Let *G* ba a topological group and let ?• $\subset G$ be an open set and ?•> is constant. Also, assume *f* : ?¢$\to$ **R** is a function.

Definition 8. The function *f* is said to be *globally* ?Ñ*Wright-convex* in ?ã if

$$f(ay^{-1}) + f(ay) \leq f(ay^{-1}z^{-1}) + f(ayz) + 2?õ \qquad (11)$$

for every $a, y, z \in$ G such that $a, ay, ay^{-1}, az^{-1}, ayz, ay^{-1}z^{-1} \in$? .

Definition 9. The function *f* is called locally ? -Wright-convex at $a \in$?& if there exists an open symmetric $V \in \Phi$ such that $aV^2 \subseteq$?& and for every $y, z \in V$ (11) is true.

Remark 4. Any globally ?J-Wright-convex function is globally ?\-midconvex .

Theorem 5. Let ?n be an open midconvex set in an a root-approximable abelian group *G*. If *f* : ?€$\to$ **R** is globally ?€-Wright-convex and locally bounded from below at a point *a* ?‘?‘, then *f* is 2?£-convex.

Proof. Suppose without loss of generality, $e \in$?‡ and $a = e$. Let $V = V^{-1} \in \Phi$ be an open neighborhood such that $V^2 \subseteq$?Õ $f(y^{-1}) + f(y) \le f(y^{-1}z^{-1}) + f(yz) + 2$?J and $f(y) \ge m$, for any $y, z \in V$ and a real number m. In (11), put $z = y^{-1}$. So

$f(y) \le 2f(e) - f(y^{-1}) + 2$?^ $\le 2f(e) - m + 2$?^

This means that f is bounded from above on V. Thus f is bounded from above at every point of ?e Consequently f is 2?e convex at every point of ?î, by Bernstein-Doetsch theorem.

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