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Shear Wave Velocity Estimation Utilizing Wireline Logs for a Carbonate Reservoir, South-West Iran

Eskandari, H.[1] , Rezaee, M.R.,[2] Javaherian, A.,[3] and Mohammadnia, M.,[4]
[1]*Amirkabir University of Technology,*
[2]*Tehran University, Faculty of Science, Geology Dept.*
[3] *Tehran University, Geophysics Institute,*
[4]*Research Institute of Petroleum Industry, NIOC,*
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Abstract
Estimation of shear wave (Vs) velocity using log data is an important approach in the seismic exploration and development of a reservoir. The velocity dispersion due to frequency difference between ultrasonic laboratory measurement on cores and low frequency is about 3.5%, and thus laboratory measurement velocities are more than sonic log in this study. Therefore, we used compressional wave velocity from sonic log to predicate shear wave velocity.

In this study, a new statistical method is presented to predict Vs from wireline log data. The model can predict shear wave velocity from petrophysical parameters and any pair of compressional wave velocity, porosity and density in carbonate rocks. The established method can estimate shear wave velocity in carbonate rocks with correlation coefficient of about 0.94.

Kaywords: *Petrophysics, acoustic properties, dispersion, sonic log, shear wave velocity, multiple regression*.

Introduction
In many developed oil fields, only compressional wave velocity may be available through old sonic logs or seismic velocity check shots. For practical purpose such as in seismic modeling, amplitude variation with offset (AVO) analysis, and engineering applications, shear wave velocities or moduli are needed. In these applications, it is important to extract, either empirically or theoretically, the needed shear wave velocities or moduli from available compressional velocities or moduli (Wang, 2000).

In rock physics and its applications, three methods are used normally to study the elastic properties of rocks: theoretical and model studies, laboratory measurement and investigations, and statistical and empirical correlations. Theoretical studies yield mathematical expressions of the elastic properties of rocks but have to make assumptions to simplify the mathematics. The assumptions are sometimes oversimplified and even unrealistic. Laboratory measurements provide data under controlled or simulated physical conditions of rocks but sample only a small portion of the reservoir. On the other hand, statistical and empirical correlations provide simple mathematical formulations of the laboratory and/or field data. These correlations often ignore the physics and complicated mathematics behind the data and require large data sets for the analyses (Wang, 2000).

The investigation of the variation in seismic velocities due to change of the dynamic reservoir properties requires controlled experiments in which accurate seismic data are complemented by detailed mineralogical and petrophysical analysis of the rock. This can only be done in a systematic way using laboratory measurements. Laboratory measurements are used to establish models to relate seismic properties (velocity and attenuation) to reservoir properties (porosity, permeability and clay content). These so-called 'petro-acoustic studies' have become increasingly an essential part of any study in reservoir geophysics (Khaksar, 2000).

During the past years, many studies have been done on elastic wave velocities focused on related petrophysical properties of rocks. Unfortunately, nearly all of these studies are about sandstone formations. In Iran most of reservoirs are carbonate rocks. In this study, we first used available data set to validate empirical correlations that have been delivered for carbonate rocks. After evolution of the effective petrophysical properties on shear wave velocity, statistical method was used to establish a correlation among effective petrophysical properties and shear wave velocitiy. The established method can estimate shear wave velocity in carbonate rocks with correlation coefficient of about 0.94.

Data sources

A data set of both compressional and shear velocities in 35 core samples (23 limestone and 12 dolomite samples). The velocities are measured at both dry and water saturated conditions. These data were gathered at an ultrasonic frequency of 0.5-1 MHz.

X-ray diffraction (XRD), thin sections and scanning electronic microscopy (SEM) are used to determine mineralogy, volume of individual minerals and other microscopic sedimentological features. The petrophysical properties of these core samples cover a wide range for exploration interest, with porosity from 0.2% to 29%, permeability from 0.02 to 228.2 mD, clay content from 0% to 10%, calcite content from 47% to 98% and dolomite from 0% to 49%.

Velocity Dispersion between Seismic and Ultrasonic Frequencies

It is known that the acoustic velocities in fluid-saturated rocks are dispersive (Khaksar, 2000). That is, the velocities are frequency dependent. The magnitude of velocity dispersion needs to be known if acoustic data obtained from laboratory measurement at ultrasonic frequencies are to be used for log analysis and seismic interpretation. Comparison of laboratory ultrasonic frequency wave propagation model of Biot (1956) and Gassmann (1951) may give estimates of total velocity dispersion between low frequency and measurement frequency. Assuming that the dry velocities are independent of frequency (Winkler, 1986), the Biot-Gassmann equations presented below are used to calculate the low frequency velocities in the fully saturated rocks. The difference between the measured velocities and the calculated low frequency velocities may be interpreted as an indication of dispersion. The following equations were used in dispersion analysis:

$$K=k_d+\{[(1-k_d/k_m)^2]/(f/k_f)+[(1-f)/k_m]-k_d/k_m^2\} \quad (1)$$

$$\mu = \mu_d \quad (2)$$

$$V_p= [(k+4\,\mu/3)/\rho]^{1/2} \quad (3)$$

$$V_s= (\mu/\rho)^{1/2} \quad (4)$$

$$\rho = f\,\rho_f + (1\text{-}f)\,\rho_m \qquad (5)$$

The parameters in equations 1 through 5 are defined as follows: K is bulk modulus; k_d , k_m and k_f are the bulk moduli of the dry rock, matrix and the pore fluid respectively; μ is the shear modulus of the saturated rock and μ_d is the shear modulus of dry rock; f is fractional porosity; ρ is bulk density, ρ_f is pore fluid density and ρ_m is grain density.

To derive the input parameters for the Gassmann equation (equ. 1) V_p and V_s measurements on dry samples were used in conjunction with density measurements to derive bulk (k_d) and shear (μ_d) moduli of the samples. Empirical relations for acoustic velocity of water at different pressure and temperatures were used to calculate the bulk modulus of water (k_f). The bulk modulus of the matrix (k_m) was determined from an extrapolation to zero porosity of the regression curve of k_d versus porosity at 30Mpa confining stress (Fig.1). Figure 2 illustrates calculated low frequency V_p considering water-wet shear modulus as input parameter for velocity calculations. Biot-Gassmann calculated velocities fit the measured ultrasonic frequency with an average of 3.5% dispersion.

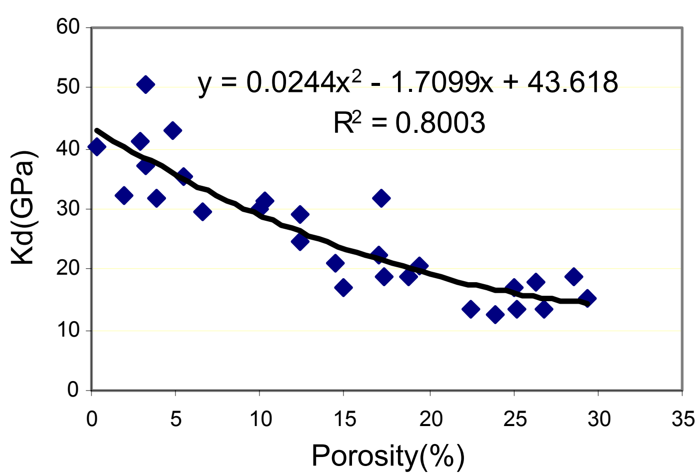

Figure 1 - Bulk modulus of air-saturated samples versus porosity at 30Mpa confining pressure. At zero porosity, k_m is equal to 43.618 Gpa.

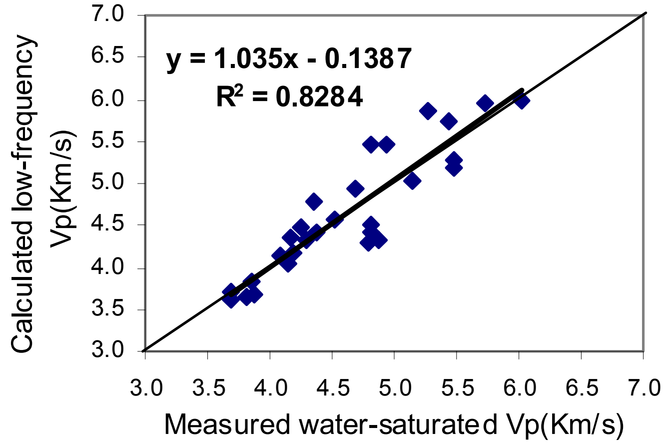

Figure 2 - Calculated low-frequency Vp using Gassmann's equation versus measured saturated Vp.

Wang and Nur (1992) commented that a 2% difference between Biot-Gassmann calculated low-frequency and measured velocities should be considered as a perfect fit for practical purposes. Due to this discrepancy between velocities measured in the laboratory at ultrasonic frequencies on water-saturated samples cannot be treated as low-frequency data and therefore we will use compressional velocities from sonic log to calculate shear velocities. As it can be expected, the difference between shear velocities measured in laboratory and calculated from Biot-Gassmann are low (Figure 3).

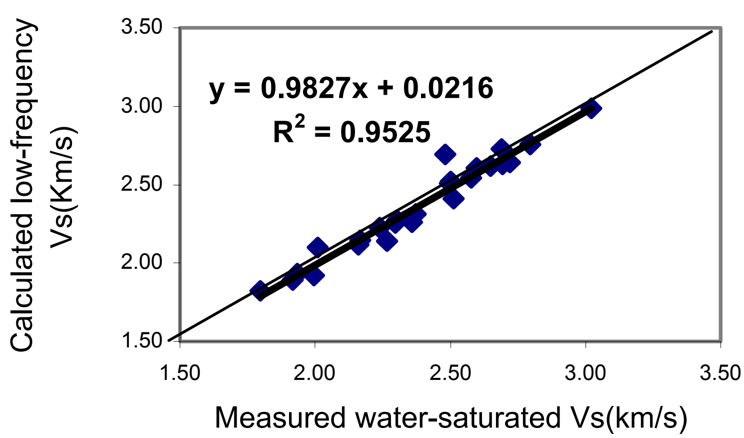

Figure 3 - Calculated low-frequency Vs using Gassmann's equation versus measured water saturated Vs.

Shear Wave Velocity Prediction

There are many applications for shear wave velocities in petrophysics, seismic and geomechanical studies. There is not any borehole in the studied field with S-wave velocity data, thus prediction of V_s from other logs was necessary. Even when an S-wave log has been run, comparison with its prediction from other logs can be a useful quality control. There are several empirical equations (for example, Han et al., (1986) and Castagna et al., (1993)) to predict V_s from other logs. In general, these empirical relationships give good result only in similar formations and their reliability for other rocks should be considered suspect until a calibration is established. It is therefore useful to have a physical model that provides some understanding of shear wave behavior (Wang, 2000).

Although, the prediction should be the same if all measurements are error free, comparison of predictions with laboratory and logging measurement show that predictions using compressional wave velocity are the most reliable especially for carbonate rocks. Figure 4 shows a good relation between V_p and V_s in well 3. Figure 5 shows a lack of relationship between velocity and core porosity for samples measured under air- and water-saturated condition under 31-33 Mpa. The significant difference in V_p/V_s values for air- and water-saturated shown in Figure 5 suggests that the velocity ratio has potential for detecting gas-saturated intervals under in-situ reservoir pressures in the study area whenever shear wave velocity data are available.

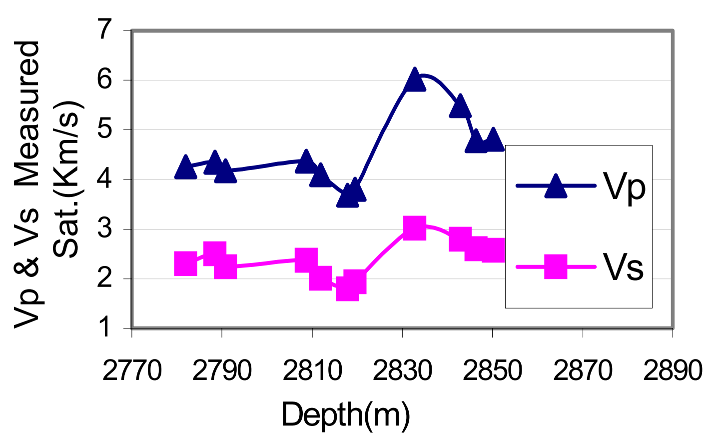

Figure 4 – A comparison between V_p and V_s in well 3, showing a good match.

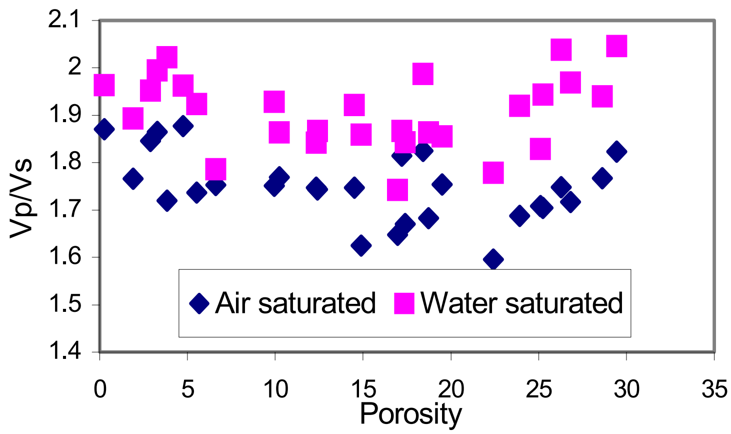

Figure 5 - Velocity ratio versus porosity at 30Mpa effective stress for air- and water-saturated.

Figure 6 shows the plots of predicted V_s, using the equation proposed by Castagna et al., (1993) versus measured data for water saturated samples from this study. Notice that the V_p data for this equation are derive from sonic log. The equations for limestone and dolomite are:

$$V_s\ (km/s) = -0.05509V_p^2+1.0168V_p-1.0305 \qquad (6)$$
$$V_s\ (km/s) = 0.583V_p-0.07776 \qquad (7)$$

Where, V_p is in km/s and derived from sonic log.

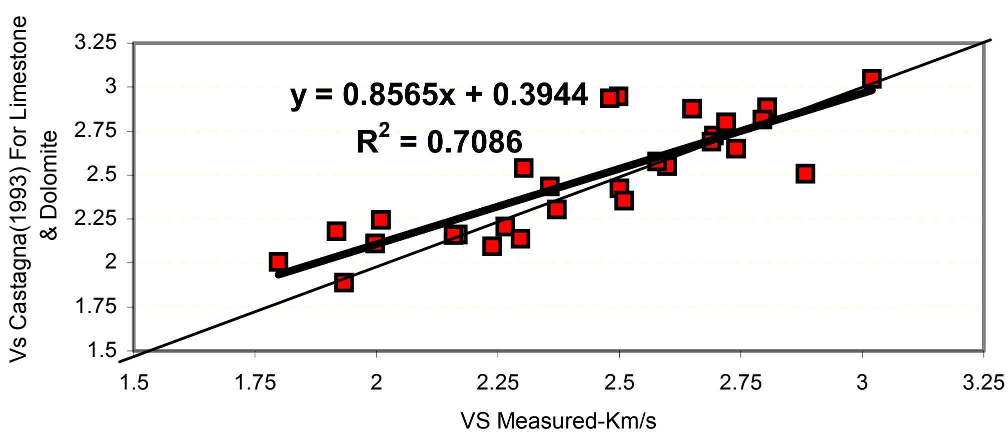

Figure 6 - Plot of calculated Vs from Castagna et al., (1993) equation versus measured Vs at 30Mpa effective stress and 100% water-saturated condition.

In order to deliver an equation with better correlation coefficient (Castagna equation has correlation coefficient of about 0.70) we used statistical method to approach a statistical correlation that calculate shear wave velocity. At first, we used only V_p from sonic log as input. In this way the best equation is as follow (equ. 8):

$$V_s\ (\mathrm{km/s}) = -0.1236V_p^2+1.6126V_p-2.3057 \qquad (8)$$

Figure 7 shows the plots of predicted V_s using the equation 8. This equation has one input parameter and correlation coefficient for this equation is approximately 0.80.

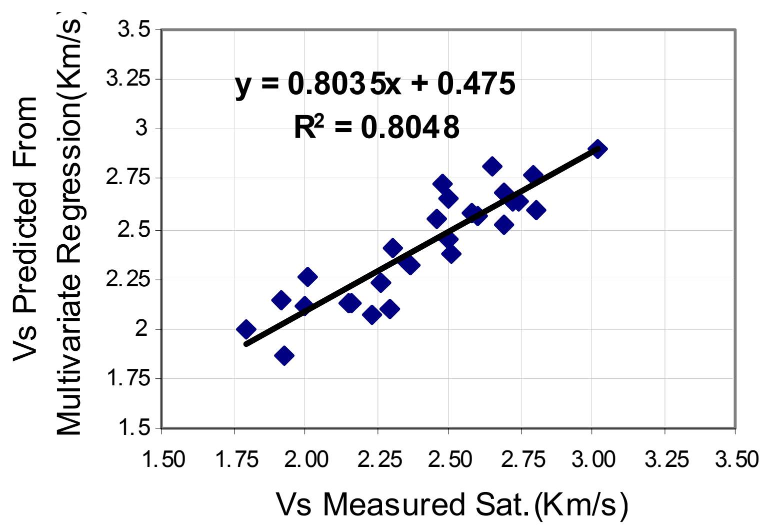

Figure 7 - Plot of predicted Vs using equation 8 versus measured Vs under 30Mpa effective stress and 100% water-saturated condition.

Other parameters including Neutron Porosity (NPHI), Bulk Density (RHOB), Gamma Ray (GR) and Deep Latrolog (LLD) were considered to include to the new equation in order to increase the accuracy of predicted V_s using multiple variable regression. Figures 8 to 10 show the effect of porosity, clay content and bulk density on V_s. Clay content is a significant factor in the study of acoustic velocities. Due to low amount of clay content in the studied samples, result suggested that clay content has a negligible effect of velocity. Shear wave velocity decreases with increase of porosity and increases with increase of bulk density.

Then these five parameters (V_p, bulk density, neutron porosity, Gamma ray and deep resistivity) were used as input of multiple regression. A multivariate model of the data solves for unknown coefficient a_0, a_1, a_2,… of multivariate equation such as equation 9:

$$V_s = a_0 + a_1 V_p + a_2 \, NPHI + a_3 \, RHOB + a_4 \, GR + a_5 \, LLD \qquad (9)$$

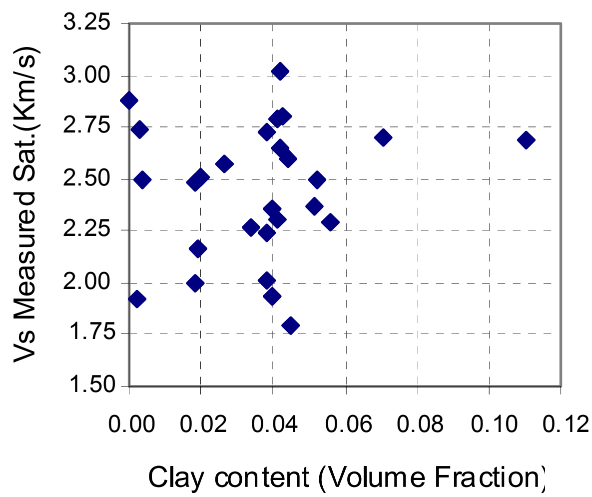

Figure 8 - Effect of clay content on shear wave velocity.

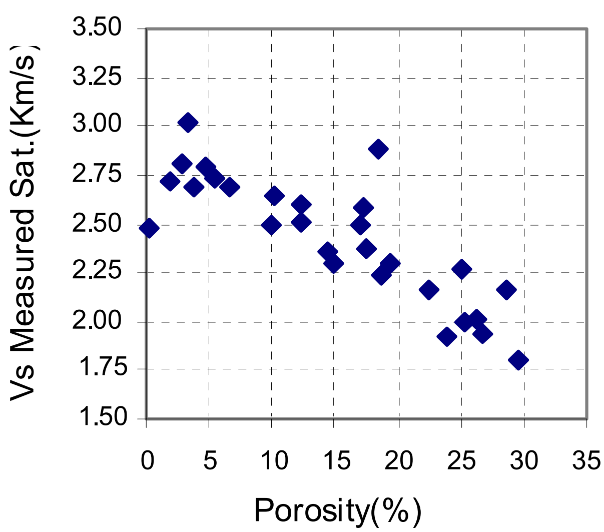

Figure 9 - Effect of porosity on shear wave velocity.

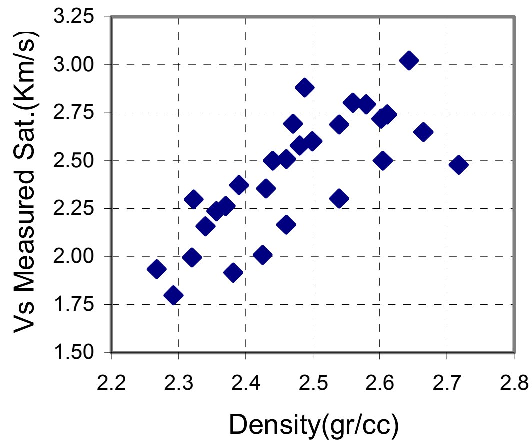

Figure 10 -Effect of bulk density on shear wave velocity.

The strength of the input variables to predict V_s is given by their degree of contribution to the V_s, which is determined by the multiple regression analysis. Contribution factors are shown in Table 1. It can be seen that the most important variables to this regression are the NPHI, RHOB and V_p that play significant roles in the model. The weakest variables are the GR and LLD. This means that they may be taken out of the model. When GR and LLD were omitted then R^2 increased from 0.91 to 0.94. These two equations are shown in Table 1 (equations a and b). Figure 11 shows a plot of the best equation for prediction of shear wave velocity with R^2 close to 0.94. This equation (equ. 10) is:

$$\mathbf{V_{s=}-17.0885 + 0.4068*Vp-2.1907*NPHI^2-1.1794*NPHI-3.2747* RHOB^2+15.3587*RHOB}$$

Table1 - Input strength for statistical equations.

Input Variable	Input Strength (Eq.a)	Input Strength (Eq.b)
A_0	3.28	-17.0885
Vp	0.4380	0.4068
NPHI	-1.3820	-1.1749, -2.1907
RHOB	-1.0544	15.3587, -3.2747
GR	0.0037	-
LLD	-0.0011	-
R^2	0.91	0.94

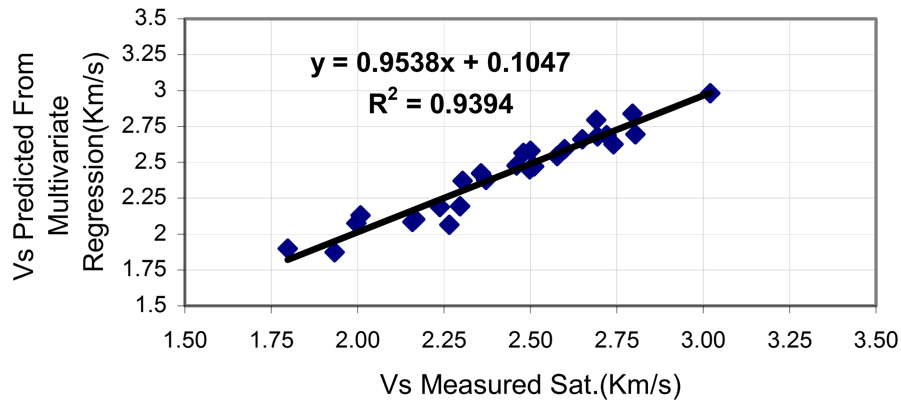

Figure 11 - Plot of predicted shear wave velocity using equation 10 versus measured data under 30Mpa effective stress and 100% water-saturated condition.

Figure 12 present the computed shear wave velocity and core shear wave velocity versus depth for well 3. Multiple regression presented robust correlation to predict shear wave velocity from well log data. Multiple regression is an extension of the regression analysis that incorporates additional independent variable in the predictive equation.

All two methods, empirical and multiple regression, were applied to log data to predict shear wave velocity for the carbonate reservoir. The result show that statistical method perform better than empirical models, which can be used only to obtain an order of magnitude for shear wave velocity.

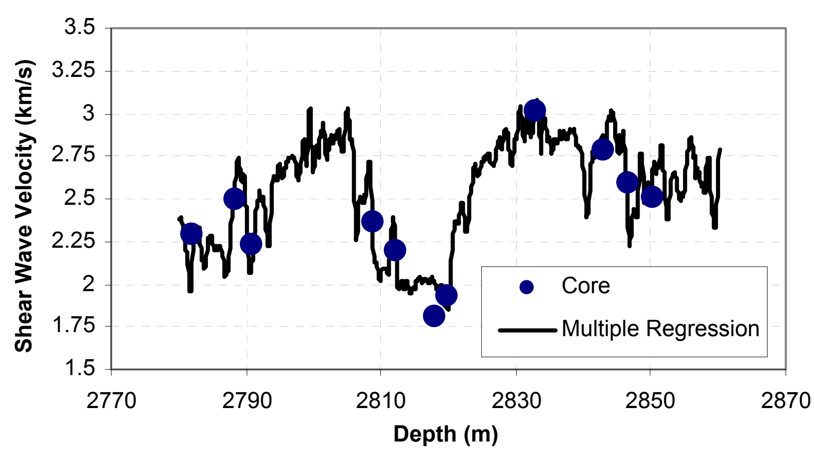

Figure 12 - Core and computed shear wave velocity for well#3.

Conclusion

This petrophysical study has investigated the use of laboratory measurements of acoustic properties on core samples for prediction of shear wave velocities using sonic log. In this study we used sonic log as input information of regression. We observed that the most important variable to this regression are the NPHI, RHOB and V_p that play significant roles in the statistical model. The introduced equation can predict shear wave velocity for carbonate reservoir with R^2 of about 0.94.

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